NAZARETH COLLEGE OF ARTS AND SCIENCE

Affiliated to University of Madras
Re-accredited by NAAC with B Grade

DEPARTMENT OF MATHEMATICS

Academic Year: 2020 – 2021

Semester: Odd

Subject Name: Dynamics

Topic Name: Kinematics

Name of the Faculty: D.Femila Jayaseeli

INTRODUCTION

Mechanics is the science which deals with the effects of forces on material bodies. Under the influence of force, a body may be in motion or in rest.

Dynamics is the science which deals with the motion of particles or bodies under the influence of forces.

Kinematics is the aspect of dynamics which deals with the motion, without reference to the forces producing it.

Statics is the science which deals with the conditions for lack of motion, under given forces.

Units

The fundamental quantities in mechanics are length, time and mass. There units are metre(m.), second(sec.) and kilogram(kg.)

Vector Quantities

Physical quantities, which have magnitudes and directions are said to be vector quantities. It is denoted by vector.

Examples:

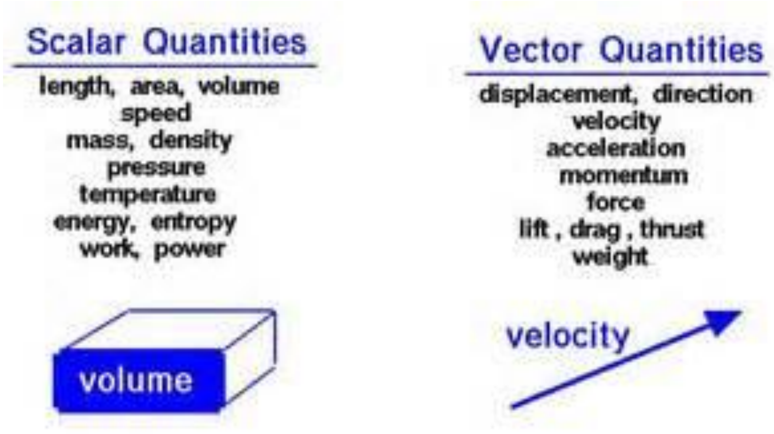
Displacement, Velocity, Acceleration and Force.

Scalar Quantities

Quantities which have only magnitudes, are scalar quantities. It is denoted by scalar.

Examples:

Length, mass, density, speed, work and energy.



Vector

The physical quantities in mechanics are vectors. It is denoted by cross.

Scalar

It is denoted by dot.

1.1 BASIC UNITS

The fundamental quantities in mechanics are length, time and mass. The following three are important systems of units

- The metre–kilogram–second system(M.K.S system)
- The centimetre-gram-second system(C.G.S system)
- The foot-pound-second system(F.P.S system)

Mass and its units

The two bodies which cause equal expansions in a place, also cause equal expansions in other places. Two such bodies seem to have something in common. This property is called the mass of the bodies.

In other words, these bodies are said to have the same mass.

The units of mass are

- Kilogram
- Pound

Kilogram : The M.K.S unit of mass is the mass of a piece of metal, arbitrarily chosen and preserved in Paris. This unit is called a kilogram.

Pound: The F.P.S unit of mass is the mass of a piece of metal preserved in London. This unit is called a pound.

Units

Quantity	M.K.S units	C.G.S units	F.P.S units
Length	Metre (m.)	cm.	Foot (ft.)
Time	Second (sec.)	sec.	Second (sec.)
Mass	Kilogram (kg.)	gm.	Pound (lb)

1.2 VELOCITY

Displacement:

When a particle moves from a point to another point, the particle is said to undergo a displacement. In other words, displacement is just a change of displacement.

Velocity

The rate of displacement, that is the rate of change of position is called the velocity of the particle. Velocity is denoted by $\vec{v}$

Acceleration

The rate of change of velocity of a particle is define as its acceleration. $\vec{f} = \frac{d}{dt}(\vec{v}) = \frac{d^2\,\vec{r}}{d^2 t}$

Speed

The magnitude of velocity, namely $\dot{s}$, is called speed. Speed is a scalar. In other words the speed of the particle is the rate at which it describes the path.

Resultant Velocity

If a particle has two velocity $\overrightarrow{v_1}$ and $\overrightarrow{v_2}$, then $\overrightarrow{v_1} + \overrightarrow{v_2}$ is said to be the resultant velocity of the particle.

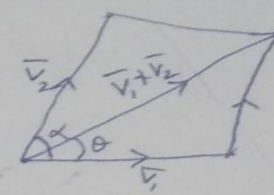
Book work 1: To find the magnitude and direction of the resultant of the velocity $\vec{v_1}$ and $\vec{v_2}$

Proof :

Now the resultant is $\bar{V}_1+\bar{V}_2$

Let the angle between $\bar{V}_1$ and $\bar{V}_2$ be α

Then the magnitude of $\bar{V}_1+\bar{V}_2$ is



$$|\bar V_1+\bar V_2| = \sqrt{(\bar V_1+\bar V_2)\cdot(V_1+V_2)} \quad [\because |\bar a| = \sqrt{\bar a\cdot\bar a}]$$

$$= \sqrt{\bar V_1\cdot\bar V_1 + V_2\cdot V_1 + V_2\cdot V_2 + V_1\cdot V_2}$$

$$= \sqrt{\bar V_1\cdot\bar V_1 + \bar V_2\cdot\bar V_2 + 2\bar V_1\cdot\bar V_2}$$

$$= \sqrt{V_1^2 + \bar V_2 + 2V_1V_2\cos\alpha} \quad [\bar a\cdot\bar b = \cos\theta\; ab]$$

where $|\bar V_1| = V_1$, $|\bar V_2| = V_2$

Let θ be the angle between $\bar V_1$ and the resultant $\bar V_1+\bar V_2$.

Then
$$\tan\theta = \frac{|\bar V_1\times(\bar V_1+\bar V_2)|}{\bar V_1\cdot(\bar V_1+\bar V_2)} \quad \left[\tan\theta = \frac{|\bar a\times\bar b|}{\bar a\cdot\bar b}\right]$$

$$= \frac{|\bar 0 + \bar V_1\times\bar V_2|}{V_1^2 + \bar V_1\cdot\bar V_2} \quad [\because \bar V_1\cdot\bar V_1 = V_1^2]$$

$$= \frac{|\bar V_1\times\bar V_2|}{V_1^2 + V_1V_2\cos\alpha}$$

$$= \frac{|V_1V_2\sin\alpha\,\hat n|}{V_1(V_1+V_2\cos\alpha)} \quad [\because |\bar a\times\bar b| = ab\sin\theta\,\hat n]$$

$$= \frac{V_1V_2\sin\alpha}{V_1+V_2\cos\alpha}$$

$$\therefore \theta = \tan^{-1}\frac{V_2\sin\alpha}{V_1+V_2\cos\alpha}.$$

Corollary 1: If $\vec{v_1}$ and $\vec{v_2}$ are equal magnitude, say v then the resultant is equally inclined at an angle $\frac{\alpha}{2}$

Proof:

WKT

$|\bar{v}_1+\bar{v}_2| = \sqrt{v_1^2+v_2^2+2v_1\cos\alpha \cdot v_2}$

$\because v_1 = v,\ v_2 = v$

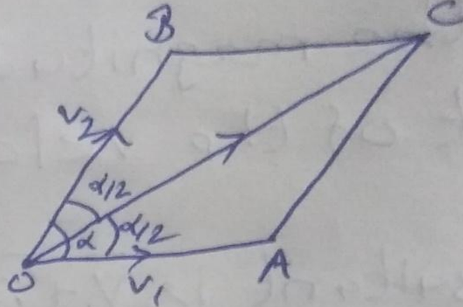
$= \sqrt{v^2+v^2+2v^2\cos\alpha}$

$= \sqrt{2(1+\cos\alpha)v^2}$

$= v\sqrt{2(2\cos^2\alpha/2)}$ $[\because 1+\cos\alpha = 2\cos^2\alpha/2]$

$= v\sqrt{4\cos^2\alpha/2}$

$= 2v\cos^2\alpha/2$



In the figure, OACB is a rhombus.

So, OC bisects $\angle AOB$.

So the resultant is equally inclined to $\bar{v}_1$ and $\bar{v}_2$ at an angle $\alpha/2$.

Corollary 2: If $\vec{v_1}$ and $\vec{v_2}$ are perpendicular to each other, then choosing $\vec{i}$ and $\vec{j}$ in their direction

Proof:

Let $\bar{V}_1 = V_1\vec{i}$, $\bar{V}_2 = V_2\bar{j}$

$\therefore |\bar{V}_1 + \bar{V}_2| = |V_1\bar{i} + V_2\bar{j}|$ $\left[\because |a\bar{i}+b\bar{j}| = \sqrt{a^2+b^2}\right]$

$= \sqrt{V_1^2 + V_2^2}$

Then $\tan\theta = \dfrac{V_2}{V_1}$ $\left[\because \tan\theta = b/a\right]$

$\theta = \tan^{-1}\dfrac{V_2}{V_1}$.

Resolution of a velocity into its components: Given the velocities $\vec{v_1}$ and $\vec{v_2}$, we have the resultant as $\vec{v_1} + \vec{v_2}$, conversely, if $\vec{v_1} + \vec{v_2}$ is given, the quantities $\vec{v_1}$ and $\vec{v_2}$ are said to be components of $\vec{v_1} + \vec{v_2}$

Bookwork 2

To resolve a velocity $\vec{v}$ into components in two given directions

Proof:

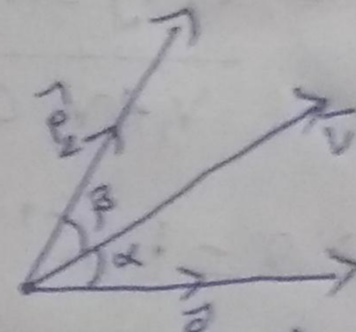
v_1

Bookwork 2:

Let $\hat{e}_1, \hat{e}_2$ be the unit vectors in the given directions.

Let them make angles α, β with $\bar{v}$.

Now $\bar{v}$ may be expressed as a linear combination of $\hat{e}_1, \hat{e}_2$ as $\bar{v} = a\hat{e}_1 + b\hat{e}_2$ —(1)



Multiplying vectorially by $\hat{e}_1$

$$\hat{e}_1 \times \bar{v} = a(\hat{e}_1 \times \hat{e}_1) + b(\hat{e}_1 \times \hat{e}_2)$$

$$v \sin\alpha \, \hat{n} = 0 + b\sin(\alpha+\beta)\hat{n}$$

where $\hat{n}$ is the unit vector perpendicular to both $\hat{e}_1, \hat{e}_2$ such that $\hat{e}_1, \hat{e}_2, \hat{n}$ form a right-handed triad.

$$\therefore v\sin\alpha \, \hat{n} = b\sin(\alpha+\beta)\hat{n}$$

$$v\sin\alpha = b\sin(\alpha+\beta)$$

$$b = \frac{v\sin\alpha}{\sin(\alpha+\beta)}$$

Eq (1) Multiplying vectorially by $\hat{e}_2$

$$\hat{e}_2 \times \bar{v} = a(\hat{e}_2 \times \hat{e}_1) + b(\hat{e}_2 \times \hat{e}_2)$$

$$v \sin\beta(-\hat{n}) = a \sin(\alpha+\beta)\,\hat{n} + b(0).$$

$$v \sin\beta(-\hat{n}) = a \sin(\alpha+\beta)\,\hat{n}$$

$$v \sin\beta = a \sin(\alpha+\beta).$$

$$\therefore a = \frac{v \sin\beta}{\sin(\alpha+\beta)}.$$

Hence $v = \dfrac{v \sin\beta}{\sin(\alpha+\beta)}\hat{e}_1 + \dfrac{v \sin\alpha}{\sin(\alpha+\beta)}\hat{e}_2$

Component of a vector in a given direction: Given a vector $\vec{v}$ and a direction specified by a unit vector $\hat{e}$, the scalar quantity $\vec{v}.\hat{e}$ is called the components of $\vec{v}$ in the direction of $\hat{e}$

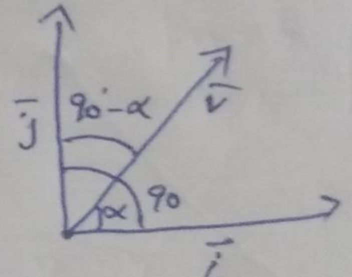
Bookwork 3: To express the velocity $\vec{v}$ in terms of its components in two perpendicular directions.

Proof:

Let $\bar{i}, \bar{j}$ be the unit vectors along the perpendicular directions.

Then, from Vector Algebra, we have

$$\bar{v} = (\bar{v}\cdot\bar{i})\bar{i} + (\bar{v}\cdot\bar{j})\bar{j}$$



If $\bar{v}$ makes angles $\alpha, 90^\circ-\alpha$ with $\bar{i}, \bar{j}$, then $\bar{a}\cdot b = ab\cos\theta$

$$\bar{v}\cdot\bar{i} = v.1.\cos\alpha$$

$$\bar{v}\cdot\bar{j} = v.1.\cos(90-\alpha)$$

$$= v\sin\alpha$$

$$\therefore \bar{v} = v\cos\alpha\,\vec{i} + v\sin\alpha\,\bar{j}$$

Problems

1. A particle has two velocities $\vec{v_1}$ anc$\vec{v_2}$. Its resultant velocity is equal to $\vec{v_1}$in magnitude. Show that, when the velocit$\vec{v_1}$ is doubled, the new resultant is perpendicular to $\vec{v_2}$

Since the magnitude of $\vec{V}_1+\vec{V}_2$ is equal to the magnitude of $\bar{V}_1$.

$\therefore |\bar{V}_1+\bar{V}_2| = |\bar{V}_1|$ (or) $|\bar{V}_1+\bar{V}_2|^2 = |\bar{V}_1|^2$

(ie) $(\bar{V}_1+\bar{V}_2)\cdot(\bar{V}_1+\bar{V}_2) = \bar{V}_1\cdot\bar{V}_1$ $[\because |a|^2 = a\cdot a]$

$\bar{V}_1\cdot\bar{V}_1 + 2\bar{V}_1\cdot\bar{V}_2 + \bar{V}_2\cdot\bar{V}_2 = \bar{V}_1\cdot\bar{V}_1$

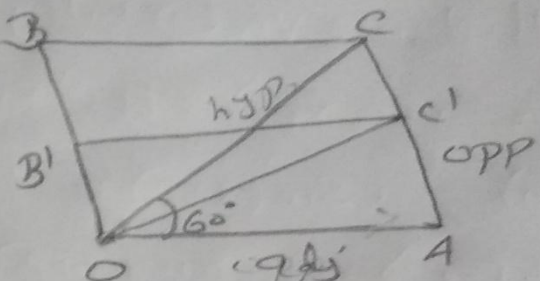
$2\bar{V}_1\cdot\bar{V}_2 + \bar{V}_2\cdot\bar{V}_2 = 0$.

$(2\bar{V}_1+\bar{V}_2)\cdot\bar{V}_2 = 0$.

So the resultant of $2\bar{V}_1$ and $\bar{V}_2$ is perpendicular to $\bar{V}_2$.

2. A particle has two velocities of equal magnitudes inclined to each other at an angle θ. If one of them is halved, the angle between the other and the original resultant velocity is bisected by the new resultant. Show that θ= 120 degree.

Let $\overline{OA}, \overline{OB}$ be the given velocities. Complete the Parallelogram OACB.



Since OA = OB, OC bisects $\angle AOB$.

Let B' be the midpoint of OB

Complete the parallelogram OAC'B'.

Now C' is the midpoint of AC.

Since OC' is the bisector of $\angle AOC$,

$$\frac{OA}{OC} = \frac{AC'}{C'C} = 1.$$

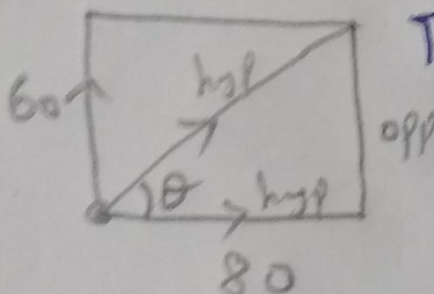
So OA = OC. But OA = OB.

Hence Δ OCA is an equilateral triangle

and $\angle AOC = 60°$.

Hence $\angle AOB = 120°$.

3. A man seated in a train whose velocity is 80kmph throws a ball horizontally and perpendicular to the train with a velocity of 60kmph. Find the velocity of the ball immediately after the throw.



The magnitude of the resultant velocity is

$= \sqrt{80^2 + 60^2}$ Kmph

$= \sqrt{10000}$ Kmph

$= 100$ Kmph.

The direction is in a horizontal plane and is inclined to the velocity of the train at angle θ is given by $\theta = \tan^{-1}\frac{60}{80}$ $\quad \tan\theta = \frac{opp}{adj}$

$\theta = \tan^{-1} 3/4$

4. A boat which can stream in still water with a velocity of 48 kmph is steaming with its bow pointed due east when it is carried by a current which flows northward with a speed of 14 kmph. Find the actual distance it would travel in 12 minutes.

Solution

Resultant speed of the vessel $= \sqrt{48^2+14^2}$ Km.p.h

$= \sqrt{2304+196}$ Km.p.h

$= \sqrt{2500}$ Km.p.h

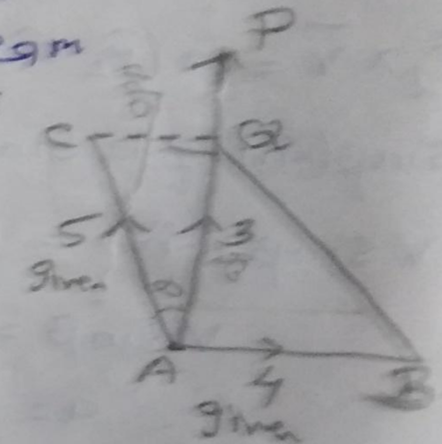
$= 50$ Km.p.h

Distance travelled in 12 minutes $= \dfrac{50 \times 12}{60}$

$= 10$ Km.

5. A stream is running at a speed of 4 kmph. Its breadth is one quarter of a mile. A man can row a boat at a speed of 5 mlph in still water. Find the direction in which he must row in order to go perpendicular to the stream and the time it takes him to cross the stream so.

Let AP be the actual path of a boat.
Let $\overline{AB}$ denote the velocity of the stream and $\overline{AC}$ denote the velocity of the boat due to rowing.



Complete the parallelogram ABQC.
Then Q will be on AP and $\overline{AQ}$ with denote the resultant velocity of the boat.

Now, $BQ^2 = AQ^2 + AB^2$

$$AQ = \sqrt{BQ^2 - AB^2} = \sqrt{AC^2 - AB}$$

$$= \sqrt{5^2 - 4^2} = \sqrt{25-16} = \sqrt{9}$$

$$AQ = 3.$$

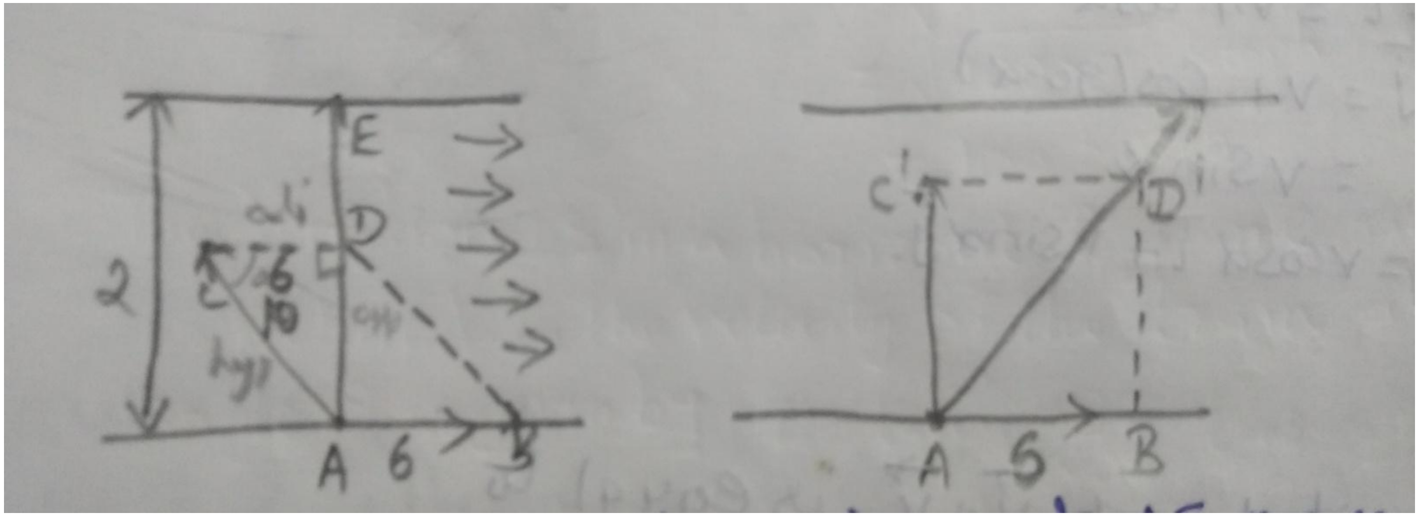
The resultant Speed of the boat is 3 m.p.h.
The time the man takes to cross the stream is $\frac{1}{4} \times \frac{1}{3} \times 60 = 5$ minutes.

$\therefore \angle CAQ \;\theta = \tan^{-1}\frac{4}{3}$ $\left[\tan\theta = \frac{opp}{hyp}\right]$

6. A boat capable of moving in still water with a speed of 10 kmph crosses a river, 2 km broad, flowing with a speed of 6 kmph. Find

(i) The time of crossing route and

(ii) The minimum time of crossing.



(i) The shortest route is the route AE perpendicular to the stream.
Let $\bar{AB}$ and $\bar{AC}$ denote respectively the velocity of the current and the velocity of the boat due to rowing so that the resultant velocity $\bar{AD}$ is along AE.

Its magnitude AD is

$$AD = \sqrt{AC^2 - CD^2}$$

$$= \sqrt{10^2 - 6^2}$$

$$= \sqrt{100 - 36}$$

$$= \sqrt{64}$$

$$= 8 \text{ Kmph}.$$

Time taken to cross the stream by the boat

is $= 2/8$ Kmph. [Speed/Dis]

$= \frac{2}{8} \times 60 = 15$ Kmpmin.

ii)

~~The time of crossing is minimum~~

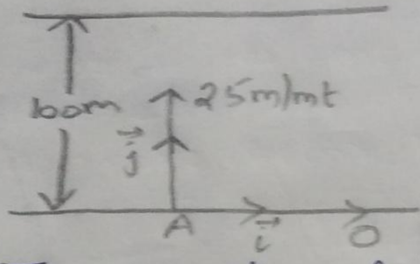
The minimum time of crossing = $\frac{\text{displacement perpendicular to the stream}}{\text{Speed in this direction}}$

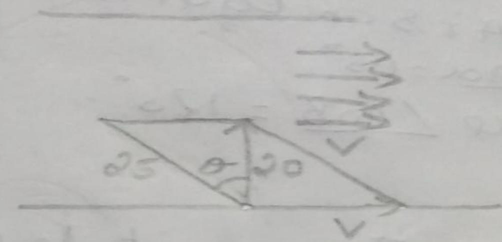
$= \frac{2}{10}$ hour.

$= \frac{2}{10} \times 60 = 12$ minutes.

7. A man can swim perpendicularly across a stream of breadth 100m in 4 minutes when there is no current and in 5 minutes when there is a downward current, Find the velocity of the current

Solution:





$S = \frac{D}{t}$

The velocity of the man in still water $= \frac{100}{4}$

$= 25$ m/mt

The velocity of the man perpendicular when there is a current is $= \frac{100}{5} = 20$ m/mt.

Taking the unit vectors $\vec{i}$ and $\vec{j}$ perpendicular vector.

we get these velocities as, $25[\cos\theta\,\vec{j} + \sin\theta(-\vec{i})]$, $v\vec{i}$.

The resultant of these two is equal to the resultant velocity of the man which is $20\vec{j}$.

$$25[-\sin\theta\,\vec{i} + \cos\theta\,\vec{j}] + v\vec{i} = -20\vec{j}$$

$$v\vec{i} + 20\vec{j} = -25[-\sin\theta\,\vec{i} + \cos\theta\,\vec{j}]$$

$$|v\vec{i} + 20\vec{j}|^2 = |25(\sin\theta\,\vec{i} + \cos\theta\,\vec{j})|^2$$

$$v^2 + 20^2 = 25^2(\sin^2\theta + \cos^2\theta)$$

$$v^2 + 20^2 = 25^2 \qquad |a\vec{i} + b\vec{j}|^2 = a^2 + b^2$$

$$v^2 = 25^2 - 20^2$$

$$= 625 - 400$$

$$v^2 = 225$$

$$v = 15$$

$\therefore$ The velocity of the current is 15 m/mt.