MEASURES OF CENTRAL TENDENCY

6.1 MEANING:

An average is a single value which is used to represent all of the values in the series. Since the average lies somewhere in between the two extremes i.e. the largest and the smallest items, it is sometimes called as a measures of central tendency.

6.2 DEFINITION:

1. According to **Croxton and Cowden.** "An average is a -single value within the range of the data that is used to represent all of the values in the series. Since an average is somewhere within the range of data, it is sometimes called measures of central value"
2. According to Clark, "An average is a figure that represents the whole group".

6.3 FUNCTIONS OF AVERAGES

According to Mooney. "The purpose of an average is to brief and to make simple representations of a group of individual value so that the brain may quickly grasp the general basis of the units of the group." Some of the main purposes and functions of Averages are as under:

(1) BRIEF DESCRIPTION:

The main purpose of an average is to present a simple and systematic description of the principal features of the raw data. As a result of it, data can be easily understood.

(2) COMPARISON:

Averages help in making comparison of different sets of data. For example a comparison of the per capita income of India and USA shows that per capita income of India is much less than the per capita income of USA. Accordingly, it is concluded that India is less developed compared to USA.

(3) FORMULATION OF POLICIES: Averages help in formulation of policies. For example, in India the per capita income is 1,11,782 per annum which is much less than many countries in the world. Accordingly, it becomes clear for the government to focus on such economic policies as are likely to increase per capita income.

(4) **STATISTICAL ANALYSIS:** Averages constitute the basis of statistical analysis. For example, if one knows the average marks secured by the students of a class in their different subjects, one can easily analyse the subjects in which the students are weak.

(5) **ONE VALUE FOR ALL:** Average represents the universe or the mass of statistical data. One value represents all values of the series. Accordingly, conclusion can be drawn in respect of the universe as a whole.

6.4 ESSENTIALS OF A GOOD AVERAGE: A good and satisfactory average should have the following features:

(1) **Clear and Stable Definition:** A good and a satisfactory average should be clear and stable in definition.

(2) **Representative:** An average value should be representative of the entire mass of data. It should be based on all the observations of the series. .

(3) **Simplicity**: Simplicity is another essential feature of a good average. It must be so simple that it is easily worked out.

(4) **Certainty**: A good average must be certain in character. Only then an average value can be used as the basis of statistical analysis.

(5) **Absolute Number**: A good average should be an absolute number. A percentage or a relative value does not serve as a good average.

(6) **Least Effect of a Change in the Sample**: An average of a series should be least affected by a change in the sample on which the average is based.

(7) **Algebraic Treatment**: A good average should be capable of further mathematical or algebraic treatment.

6.5 TYPES OF STATISTICAL AVERAGES:

Averages are broadly classified into two categories:.

(1) Mathematical Averages (2) Positional Averages

Following chart reveals their further categorization:

Averages	→	Mathematical Averages	→	Arithmetic Mean Geometric mean Harmonic Mean
	→	Positional Averages	→	Median Partition Value Mode

6.6 ARITHMETIC MEAN:

Arithmetic Mean is a simple average of all items in a series. It is the simplest measure of central tendencies. The arithmetic mean of a series is simply called 'Mean'. If, for example, Ram plays 5 matches, Shyam 6, Mohan 7, Kamal 8 and Ravi 9-matches a week, the average number of matches played by Ram, Shyam, Mohan, Kamal and Ravi would be-determined as under:

No. of Matches = 5+6+7+8+9 = 35

No of Boys = 5

$$\text{Mean} = \frac{\text{Total Value of the Items}}{\text{No.of Items}} = \frac{35}{5} = 7$$

DEFINITION:

Arithmetic Mean or Mean is the number which is obtained by adding the values of all the items of a series and dividing the total by the number of items

According to H. Secrist, "The arithmetic mean is the amount secured by dividing the sum of value of the items in a series by their numbers."

6.7 PROPERTIES OF ARITHMETIC MEAN (AM):

1. The sum of deviations of the items from AM is always Zero.
2. The sum of squared deviation of the items from AM is minimum
3. If each item of a series in increased, decreased, multiplied or divided by some constant then AM also increases, decreases, multiplies or is divided by the same constant.
4. The product of the AM and the no. of items on which mean is based is equal to the sum of all given items.
5. If each item of the original series is replaced by the actual mean, then the sum of these substitutions will be equal to the sum of the individual items.

6.8 MERITS OF ARITHMETIC MEAN:

The following are some of the main merits of arithmetic mean:

1. **Simplicity:** From the viewpoint of calculation and usage, arithmetic mean is the simplest of all the measures of central tendency.
2. **Certainty:** Arithmetic Mean is a certain value; it has no scope for estimated values.
3. **Based on All Items:** Arithmetic Mean is based on all the items in a series. It is, therefore, a representative value of the different items.
4. **Stability:** Arithmetic Mean is a stable measure of central tendency. This is because changes in the sample of series have minimum effect on the arithmetic average.
5. **Basis of Comparison:** Being stable and certain, Arithmetic Mean can be easily used for comparison.
6. **Accuracy Test:** Arithmetic Mean can be tested for its accuracy as a representative value of the series.

DEMERITS OF ARITHMETIC MEAN:

Arithmetic Mean suffers from following demerits:

(1) **Effect of Extreme Value:** The main defect of arithmetic mean is that it gets distorted by extreme values of the series. Therefore, it is not always an accurate measure. To illustrate, pocket expenditure of a rich student may be if 2,000, while his four friends incur pocket expenditure of if Rs.100, Rs. 80, Rs.70 and Rs.50 respectively. The average pocket expenditure of all the five students would be:

$$X = \frac{(2{,}000+100+80+70+50)}{5} = \frac{2{,}300}{5} = \text{Rs.}460$$

Certainly this is not such an accurate mean of the pocket expenditure of five students as 4 out of 5 students incur pocket expenditure just ranging between Rs. 50 to Rs. 100. The mean value of Rs. 460 is largely owing to the fact that there is an extreme value of Rs. 2,000 in the series. Such a mean is certainly not a representative value of all the items in the series.

(2) **Mean Value may not figure in the series at all.** The mean value may sometimes be that value which does not figure in the series at all.

The average of 2, 3 and 7 is $\frac{(2+3+7)}{3} = 4$ which is not there in the series.

It Further Erodes its representative Character.

(3) **Mean is not useful for studying the qualitative phenomena eg.,** beauty, honesty, intelligence, etc.

TYPES OF ARITHMETIC MEAN:

ARITHMETIC MEAN IS OF TWO TYPES:

1. Simple Arithmetic mean: In it, all items of a series are given equal importance.
2. Weighted Arithmetic mean: In it different items of a series are accorded with different weights in accordance with their relative importance.

6.9 METHODS OF CALCULATING SIMPLE ARITHMETIC MEAN:

We know, there are three types of statistical series:

1. Individual series,
2. Discrete Frequency Series, and
3. Frequency Distribution or continuous series

Arithmetic mean may be calculated with respect to these series using different method as discussed below:

6.10 CALCULATION OF SIMPLE ARITHMETIC MEAN IN CASE OF INDIVIDUAL SERIES:

In the case of individual series, Arithmetic mean may be calculated by two methods:

1. Direct Method, or
2. Short – cut Method

1. DIRECT METHOD:

Following steps are involved in this method:

1. Add up values of all the items of a series, $(\overline{X})$
2. Find out total number of items in the series (N); and
3. Divide the total of value $(\overline{X})$ of all the items with the number of items (N). Thus,

$$\overline{X} = \frac{\overline{X}}{N} \text{ or } \overline{X} = \frac{\text{Total value of the Items}}{\text{No.of Items}}$$

PROBLEM: 1

Pocket allowance of 10 students is 15,20,30,22,25,18,40,50,55 and 65, Find out the average pocket allowance.

Solution:

Pocket Allowance (In Rs)
15
20
30
22
25
18
40
50
55
65
$\overline{X} = 340$

$$\overline{X} = \frac{\overline{X}}{N} \text{ or } = \frac{X_1 + X_2 + \ldots\ldots\ldots X_{10})}{10} = \frac{340}{10} = 34$$

Or, Average pocket allowance of the 10 students. = **34**

FORMULA:

$$\overline{X} = \frac{\Sigma fX}{\Sigma f}$$

PROBLEM: 3

Following is the weekly wage earnings of 19 workers.

Wages	10	20	30	40	50
No of workers	4	5	3	2	5

Calculate arithmetic mean using direct method.

Solution:

Wages (X)	**No of workers or Frequency (f)**	**Multiple of the value of X and Frequency (fx)**
10	4	4 x 10 = 40
20	5	5 x 20 = 100
30	3	3 x 30 = 90
40	2	2 x 40 = 80
50	5	5 x 50 = 250
	$\Sigma f = 19$	$\Sigma fX = 560$

$$\overline{X} = \frac{\Sigma fX}{\Sigma f} = \frac{560}{19} = 29.47$$

Mean wages earning of 19 workers = **29.47**

(I) DIRECT METHOD:

This method involves the following steps:

(i) The mid-values of the class intervals are calculated. These may be indicated by 'm'. To find the mid-value of class intervals, the lower and upper limits of that class are added and then divided by '2'. Thus, mid-value of the class 10-20 would be determined as:

$$M = \frac{l_1 + l_2}{2} = \frac{10+20}{2} = \frac{30}{2} = 15$$

Here, m = mid - value; l_1= lower limit of the class; l_2 = upper limit of the class.

(ii) Mid-values are multiplied by their corresponding frequencies. The multiples fm are added up to get $\sum fm$.

(iii) $\sum fm$ is divided by $\sum f$. The resultant value would be the mean value.

FORMULA:

$$\overline{X} = \frac{\sum fm}{\sum f}$$

PROBLEM: 6

The following Table shows marks in Statistics secured by students of B.com- gen in your college in their examination. Calculate mean ' marks using' Direct Method.

Marks	0-10	10-20	20-30	30-40	40-50
No. of Students	20	24	40	36	20

Solution:

Marks	Mid – value $(m = \frac{l_1 + l_2}{2})$	No.of Students or Frequency (f)	Multiple of Mid-value and Frequency (fm)
0-10	$\frac{0+10}{2} = 5$	20	20 x 5 = 100
10 -20	$\frac{10+20}{2} = 15$	24	24 x 15 =360
20 - 30	$\frac{20+30}{2} = 25$	40	40 x 25 = 1,000
30 – 40	$\frac{30+40}{2} = 35$	36	36 x 35 = 1,260
40 - 50	$\frac{40+50}{2} = 45$	20	20 x 45 = 900
		$\sum$ f=140	$\sum$ fm=3,620

$$\overline{X} = \frac{\sum fm}{\sum f}$$

$$= \frac{3,620}{140}$$

$$= 25.86$$

Arithmetic Mean = 25.86 marks.

Thus, Mean Marks = **25.86**

PROBLEM: 9

Marks in statistics of the students of B.com are given below. Find out Arithmetic Mean.

Marks	No. of Students
Less than 10	5
Less than 20	17
Less than 30	31
Less than 40	41
Less than 50	49

Solution:

A cumulative Frequency Distribution should first be converted into a Simple Frequency Distribution, as under:

Conversion of a Cumulative Frequency Distribution into a Simple Frequency Distribution

Marks	No. of Students
0-10	5
10-20	17-5=12
20-30	31-17=14
30-40	41-31=10
40-50	49-41=8

Now, mean value of the data is obtained using Direct Method as under:

Calculation of Mean

Marks (X)	Mid – value ($m = \frac{l_1 + l_2}{2}$)	Frequency (f)	Multiple of Mid-value and Frequency (fm)
0-10	5	5	25
10-20	15	12	180
20-30	25	14	350
30-40	35	10	350
40-50	45	8	360
		$\sum f=49$	$\sum fm=1,265$

$$\bar{X} = \frac{\sum fm}{\sum f}$$

$$= \frac{1,265}{49}$$

$$= 25.82$$

Arithmetic mean = **25.82**

PROBLEM: 10

The following table shows marks in economics of the students of a class. Calculate Arithmetic Mean.

Marks	No. of Students
More than 0	30
More than 2	28
More than 4	24
More than 6	18
More than 8	10

Solution:

Converting cumulative frequency Distribution into a Simple Frequency Distribution, we get the following.

Marks	No. of Students
0-2	30-28=2
2-4	28-24=4
4-6	24-18=6
6-8	18-10-8
8-10	10-0=10

Arithmetic Mean of this continuous series is estimated below, using Direct Method.

Calculation of Arithmetic Mean using Direct Method

Marks	Mid-value (m)	Frequency (f)	Multiple of Mid-value and Frequency (fm)
0-2	1	2	2
2-4	3	4	12
4-6	5	6	30
6-8	7	8	56
8-10	9	10	90
		$\sum f=30$	$\sum fm=190$

$$\overline{X} = \frac{\sum fm}{\sum f}$$

$$= \frac{190}{30}$$

$$= 6.33$$

Thus, mean marks = **6.33**

CALCULATION OF ARITHMETIC MEAN IN A MID- VALUE SERIES

PROBLEM: 11

Following Table gives marks in statistics of the students of a class. Find out mean marks.

Mid-value	5	10	15	20	25	30	35	40
No. of Students	5	7	9	10	8	6	3	2

Solution:

In this series, mid-values are already given. The calculation of Arithmetic Mean involves the same procedure as in the case of exclusive series.

Calculation of Arithmetic Mean

Marks	No. of students or frequency (f)	Multiple of Mid-value and Frequency (Fm)
5	5	25
10	7	70
15	9	135
20	10	200
25	8	200
30	6	180
35	3	105
40	2	80
	$\sum f=50$	$\sum fm=995$

$$\overline{X} = \frac{\sum fm}{\sum f}$$

$$= \frac{995}{50}$$

$$= 19.9$$

Arithmetic mean = **19.9 Marks.**