NAZARETH COLLEGE OF ARTS AND SCIENCE

DEPARTMENT OF MATHEMATICS

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Mathematics I

Topic: Binomial Theorem

Sec 1.1.1: SUMMATION OF SERIES USING BINOMIAL THEOREM

BINOMIAL SERIES

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \cdots + \frac{n(n-1)\dots(n-r+1)}{r!}x^r + \cdots \quad \dots(1)$$

Some cases of Binomial Expansion

1. $(1-x)^n = 1 - nx + \frac{n(n-1)}{2!}x^2 - \frac{n(n-1)(n-2)}{3!}x^3 + \cdots \quad \dots(2)$
2. $(1-x)^{-n} = 1 + nx + \frac{n(n+1)}{2!}x^2 + \frac{n(n+1)(n+2)}{3!}x^3 + \cdots \quad \dots(3)$
3. $(1+x)^{-n} = 1 - nx + \frac{n(n+1)}{2!}x^2 - \frac{n(n+1)(n+2)}{3!}x^3 + \cdots \quad \dots(4)$

When $n = \frac{p}{q}$, then

4. $(1+x)^{\frac{p}{q}} = 1 + \frac{p}{q}x + \frac{\frac{p}{q}\left(\frac{p}{q}-1\right)}{2!}x^2 + \frac{\frac{p}{q}\left(\frac{p}{q}-1\right)\left(\frac{p}{q}-2\right)}{3!} +$

   $= 1 + \frac{p}{1!}\left(\frac{x}{q}\right) + \frac{p(p-q)}{2!}\left(\frac{x}{q}\right)^2 + \frac{p(p-q)(p-2q)}{3!}\left(\frac{x}{q}\right)^3 + \cdots \quad \dots(5)$
5. $(1-x)^{\frac{p}{q}} = 1 - \frac{p}{1!}\left(\frac{x}{q}\right) + \frac{p(p-q)}{2!}\left(\frac{x}{q}\right)^2 - \frac{p(p-q)(p-2q)}{3!}\left(\frac{x}{q}\right)^3 + \cdots \quad \dots(6)$
6. $(1+x)^{-\frac{p}{q}} = 1 - \frac{p}{1!}\left(\frac{x}{q}\right) + \frac{p(p+q)}{2!}\left(\frac{x}{q}\right)^2 - \frac{p(p+q)(p+2q)}{3!}\left(\frac{x}{q}\right)^3 + \cdots \quad \dots(7)$
7. $(1-x)^{-\frac{p}{q}} = 1 + \frac{p}{1!}\left(\frac{x}{q}\right) + \frac{p(p+q)}{2!}\left(\frac{x}{q}\right)^2 + \frac{p(p+q)(p+2q)}{3!}\left(\frac{x}{q}\right)^3 + \cdots \quad \dots(8)$

Problem

1. *Sum the series* $\mathbf{1+\frac{3}{4}+\frac{3.5}{4.8}+\frac{3.5.7}{4.8.12}+}$

Solution: Let $S = 1+\frac{3}{4}+\frac{3.5}{4.8}+\frac{3.5.7}{4.8.12}+\cdots$

$$= 1+\frac{3}{1}\left(\frac{1}{4}\right)+\frac{3.5}{1.2}\left(\frac{1}{4}\right)^2+\frac{3.5.7}{1.2.3}\left(\frac{1}{4}\right)^3+\cdots$$

$$S = 1+\frac{3}{1!}\left(\frac{1}{4}\right)+\frac{3.5}{2!}\left(\frac{1}{4}\right)^2+\frac{3.5.7}{3!}\left(\frac{1}{4}\right)^3+\cdots \qquad \text{..............................} (1)$$

We know that,

$$\boxed{(1-x)^{-\frac{p}{q}} = 1+\frac{p}{1!}\left(\frac{x}{q}\right)+\frac{p(p+q)}{2!}\left(\frac{x}{q}\right)^2+\frac{p(p+q)(p+2q)}{3!}\left(\frac{x}{q}\right)^3+\cdots}$$

.... (2).

Comparing (1) & (2) we get

$$p = 3$$

$$p+q=5$$
$$3+q=5$$
$$q=5-3$$
$$q=2$$

$$\frac{x}{q}=\frac{1}{4}$$
$$\frac{x}{2}=\frac{1}{4}$$
$$x=\frac{2}{4}$$
$$x=\frac{1}{2}$$

$$\frac{p}{q}=\frac{3}{2}$$

$$S = (1 - x)^{-\frac{p}{q}}$$

$$= \left(1 - \frac{1}{2}\right)^{-\frac{3}{2}}$$

$$= \left(\frac{1}{2}\right)^{-\frac{3}{2}} = (2)^{\frac{3}{2}}$$

$$S = 2\sqrt{2}$$